CSI 972 Follow-Up Lecture Notes, August 27

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- The question was raised whether it would make sense to build the basic underlying structures of probability theory on closed sets instead of open sets. I replied that I thought so, but there might be some "gotcha" along the way. Note that Exercise 4 asks you to show that the standard Borel σ-field can equivalently be based on open sets, half-open sets, or closed sets. I still have not been able to think of any problems that could arise following a development based on closed sets. Especially in light of the equality of $\sigma(\mathcal{C})$, $\sigma(\mathcal{D})$, and $\mathcal{B}$, where $\mathcal{C}$ is the collection of half-open intervals and $\mathcal{D}$ is the collection of closed intervals, I don't think problems would arise.

  (For your solution of Exercise 4, you should note that there are two parts (the two "equals" signs). Your solution should be very clear; work one part at a time. This same comment applies to many of the exercises in this class; they are multi-part. You must clearly state what you are trying to prove in your homework.)

- My example showing that the union of two σ-fields is not necessarily a σ-field was correct. I was just standing "too close to the board" to see it. (If you've ever lectured at the board, you know what I mean.) The point is, a union of two members of the union of the σ-fields are not in the union.

  Just to be clear, I'll restate the example:

  Let

  $$\Omega = \{a, b, c\},$$

  $$\mathcal{F}_1 = \{\emptyset, \{a\}, \{b, c\}, \Omega\},$$

  and

  $$\mathcal{F}_2 = \{\emptyset, \{b\}, \{a, c\}, \Omega\}.$$

  These are both clearly σ-fields. Now, consider

  $$\mathcal{F}_1 \cup \mathcal{F}_2 = \{\emptyset, \{a\}, \{b\}, \{b, c\}, \{a, c\}, \Omega\}.$$

  This does not contain the union of two of its members,

  $$\{a\} \cup \{b\} = \{a, b\},$$

  so $\mathcal{F}_1 \cup \mathcal{F}_2$ is not a σ-field.

You should read through the full chapter in Shao, paying particular attention to the facts about integrals on pages 12–14. Also, I did not discuss some important facts about derived distributions, such as Cochran's theorem (p. 27). Because I plan to stick very closely to the book, but obviously I cannot mention everything in the book, you should read the book carefully.

We will begin next week with Section 1.4, but I will discuss questions on the previous sections if necessary.